Image Segmentation through Denoising

Valentina Staneva, vstaneva@lanl.gov
Mentor: Rick Chartrand, rickc@lanl.gov

Introduction

Extraction of shapes from images of physical experiments is very important for research at Los Alamos National Laboratory. Achieving a truthful segmentation of the shapes from the background usually requires first denoising of the images, and then applying a specific segmentation algorithm which can range from simple thresholding to complex level set segmentation. In our work we modify a standard model for image denoising, which results in automatic segmentation of the image without having an exta step. We accomplish this through forcing the model not only to remove the noise from the image but also to strongly emphasize the edges in it.

Total Variation Denoising and Segmentation

Two of the basic problems in image processing are denoising and segmentation. They are closely related, with similar objectives: given a noisy image, return a noise-free image while preserving important information from the original. Perhaps the greatest amount of information in an image is contained in the edges of objects. Consequently, preserving edges in an image is of paramount importance for both denoising and segmentation. One can regard the two problems as differing only in degree: in denoising, one seeks to remove noise and as little else as possible, while in segmentation, the goal is to remove all variation except for the edges of image regions.

Thus it is not surprising that there are models that are used for both problems. The best-known example of this is the Mumford-Shah functional [1]. When the image we are trying to reconstruct is assumed to be piecewise constant and its values are restricted to 0 and 1 this functional becomes equivalent to the restriction to binary functions of Rudin-Osher-Fatemi total variation denoising model [2]:

$$F(u) = \int_\Omega |\nabla u| + \lambda \int_\Omega |u-d|^2. \tag{1}$$

Here d is the noisy data, u is the reconstructed image, and the minimizer of the functional is the actual solution. λ is the regularization parameter which balances the relative effect of the two terms.

In our work, we simultaneously modify the regularization of edges and the image away from the edges, by introducing a small exponent $p>0$:

$$F_p(u) = \frac{1}{p}\int_\Omega |\nabla u|^p + \lambda \int_\Omega |u-d|^2. \tag{2}$$

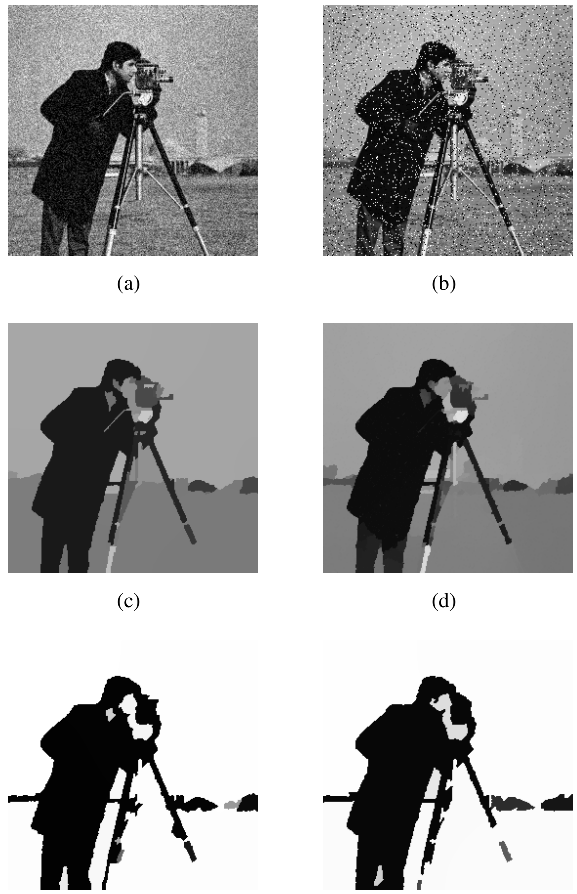
This has two effects. First, small but nonzero values of $|\nabla u|$ are penalized more, which makes the minimizer tend to be piecewise constant. We thus obtain segmentation without explicitly assuming a particular form for u. (This is the same phenomenon as the well known "staircasing" artifact of the ROF model, taken to a greater degree.) Second, as explained in [3], $\int |\nabla u|^p$ places much weaker penalty on edges, than that of total variation. This allows the segmentation to capture the boundaries of complicated regions more accurately.

Solution and Results

$F_p(u)$ is non-convex in this case, so the existence and uniqueness of a minimizer are not guaranteed. That is why this model has not been thouroughly explored. Surprisingly, in practice, the lack of convexity does not seem to cause problems. In order to find a minimum of the functional in (2) we use a straightfoward generalization of the fixed-point method of Vogel and Oman [4]. We apply the algorithm with $p=0.01$ to the cameraman image corrupted by Gaussian noise with standard deviation of 0.1 (Figure 1(a)). For an initial case we first use the noisy image. The result we obtain after 20 iterations is displayed in

Figure 1(c). Although the initial image is a complex grayscale image, which even without noise is a difficult segmentation problem, the final image consists of only a few grayscale levels. The number of the final regions and the crudeness of their boundaries can be regulated through the regularization parameter λ. If we want the final image to be close to a binary image, we can implement the algorithm using as a starting point an image obtained by thresholding the noisy image. The result only after 10 iterations has even fewer grayscale levels (Figure 1(e)). The observation that different starting points lead to different final results shows the existence of local minima. This allows us to vary the segmentation of the image, which will reflect properties of the initial guess.

The performance of the method does not depend exclusively on having an L^2-norm as a data fidelity term. For example, we can implement it with an L^1-norm, known for removing salt-and-pepper noise. On the cameraman image in Figure 1(b), in which 10% of the pixels have been corrupted by salt and pepper noise, the algorithm exhibits the same behavior. Results with two different starting points are shown in Figures 1(d) and 1(f). A Poisson noise data fidelity term can also be fit into the model. This flexibility of the algorithm makes it applicable to a wide variety of segmentation problems.



(a) (b) (c) (d) (e) (f)

(a) The cameraman image corrupted by Gaussian noise; (b) the cameraman image corrupted by salt and pepper noise; (c) segmentation of (a) obtained with the noisy image as an initial guess; (d) segmentation of (b) obtained with the noisy image as an initial guess; (e) segmentation of (a) obtained with the thresholded image as an initial guess; (f) segmentation of (b) obtained with the thresholded image as an initial guess;.

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